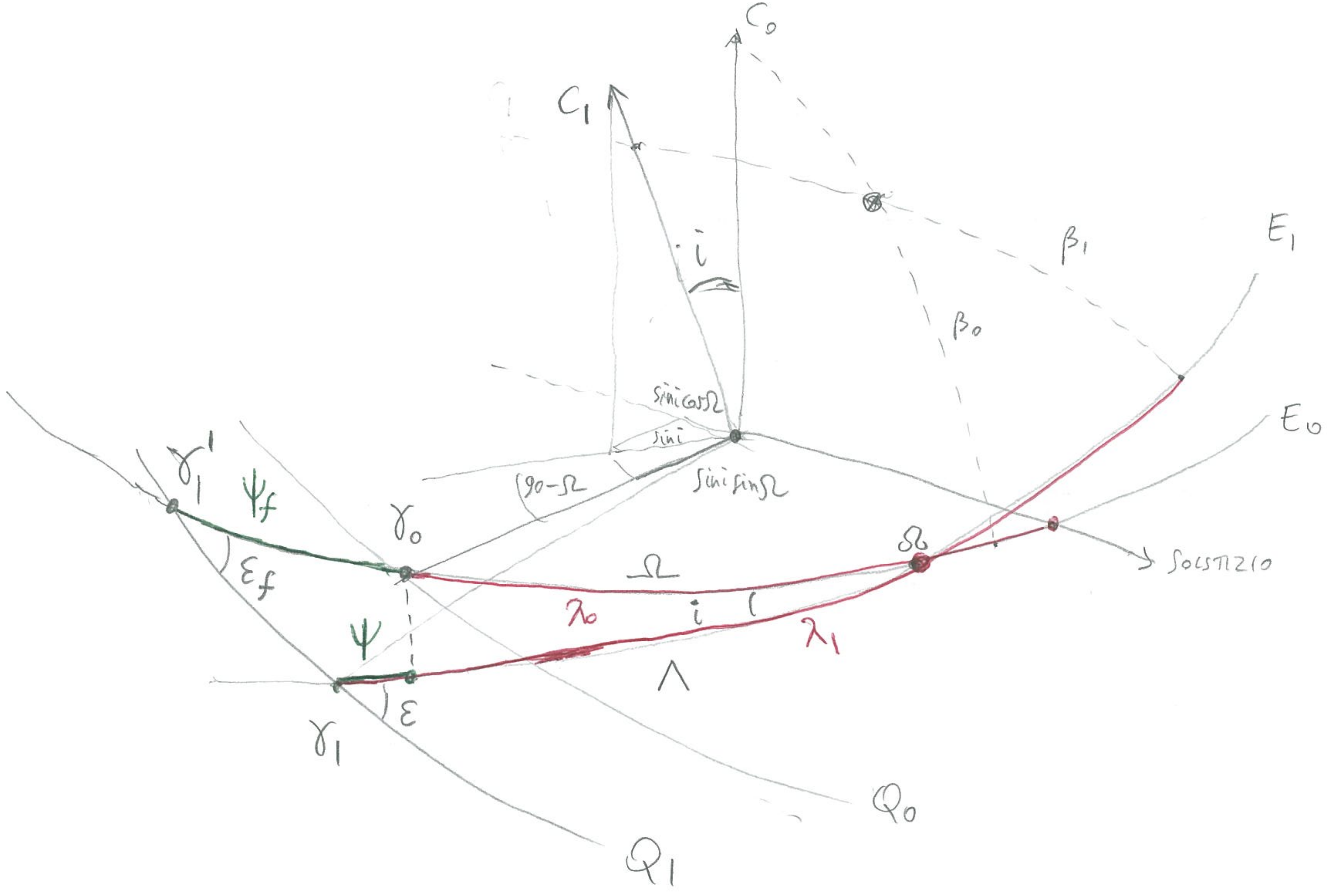
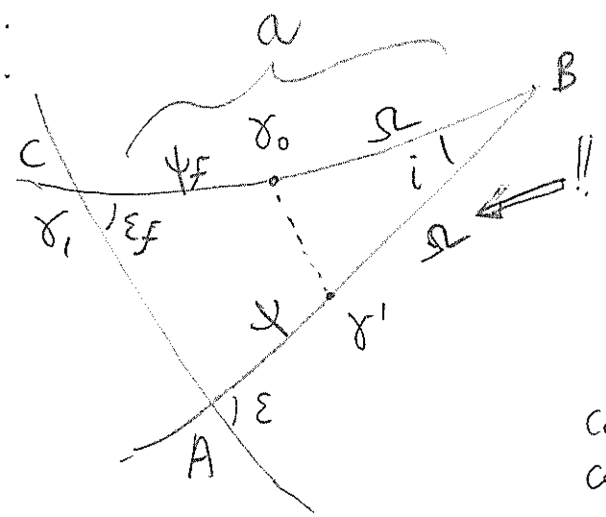


C0
C1
i
β1
β0
E1
E0
sin i cos Ω
sin i
sin i sin Ω
90−Ω
γ'1
ψf
γ0
εf
Ω
☊
SOLSTIZIO
λ0
i
λ1
ψ
Λ
γ1
ε
Q0
Q1



Formule di Berger (1976) (11) e (12)

$$\boxed{a = \psi_f + \Omega \qquad C' = \varepsilon_f \qquad B = i} \qquad \text{inoltre } A = 180° - \varepsilon$$

Berger dà per scontato che $\boxed{c = \psi + \Omega}$

$$\cos a \cos C' = \cot b \sin a - \cot B \sin C' \qquad (\mathrm{I})$$

$$\cos a \cos B = \cot c \sin a - \cot C' \sin B \qquad (\mathrm{II})$$

da (I): $\cot b \sin a = \cos a \cos C' + \cot B \sin C'$ $\qquad$ inoltre $\dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b} \rightarrow \sin^2 A = \dfrac{\sin^2 B}{\sin^2 b} \sin^2 a$

cioè $\cot b = (\cos a \cos C' + \cot B \sin C') / \sin a$

$$\sin^2 A = \frac{\sin^2 B}{\sin^2 b} \sin^2 a = \sin^2 B \sin^2 a \left(1 + \cot^2 b\right) =$$

$$= \sin^2 B \sin^2 a \left[1 + \frac{(\cos a \cos C' + \cot B \sin C')^2}{\sin^2 a}\right] =$$

$$= \sin^2 B \cancel{\sin^2 a} \left(\frac{\sin^2 a + \cos^2 a \cos^2 C' + \cot^2 B \sin^2 C' + 2 \cos a \cos C' \cot B \sin C'}{\cancel{\sin^2 a}}\right)$$

$$= \sin^2 B \left(\sin^2 a + \cos^2 a \cos^2 C'\right) + \cancel{\sin^2 B} \frac{\cos^2 B}{\cancel{\sin^2 B}} \sin^2 C' + \sin 2C' \cos a \sin^{\cancel{2}} B \frac{\cos B}{\cancel{\sin B}}$$

$$= \sin^2 B \left(\sin^2 a + \cos^2 a \cos^2 C'\right) + \cos^2 B \sin^2 C' + \sin 2C' \cos a \frac{\sin 2B}{2}$$

OK (11)

da (II): $\cot c \sin a = \cos a \cos B + \cot C' \sin B$

$$\cos c \sin a = \sin c \left(\cos a \cos B + \cot C' \sin B \right) = \sin c \cos a \cos B + \sin c \frac{\cos C'}{\sin C'} \sin B$$

perché $\dfrac{\sin c}{\sin C'} = \dfrac{\sin b}{\sin B} = \dfrac{\sin a}{\sin A}$

$$\cos c \sin a = \sin c \cos a \cos B + \cos C' \sin B = \frac{\sin a}{\sin A} \sin C' \cos a \cos B + \frac{\sin a}{\sin A} \cos C' \sin B$$

$\boxed{\psi_f - \psi = a - c}$ e $\sin(a-c) = (\sin a \cos c) - \cos a \sin c$

$$\sin(a-c) = \frac{\sin a}{\sin A}\left(\sin C' \cos a \cos B + \cos C' \sin B \right) - \cos a \frac{\sin a}{\sin A} \sin C'$$

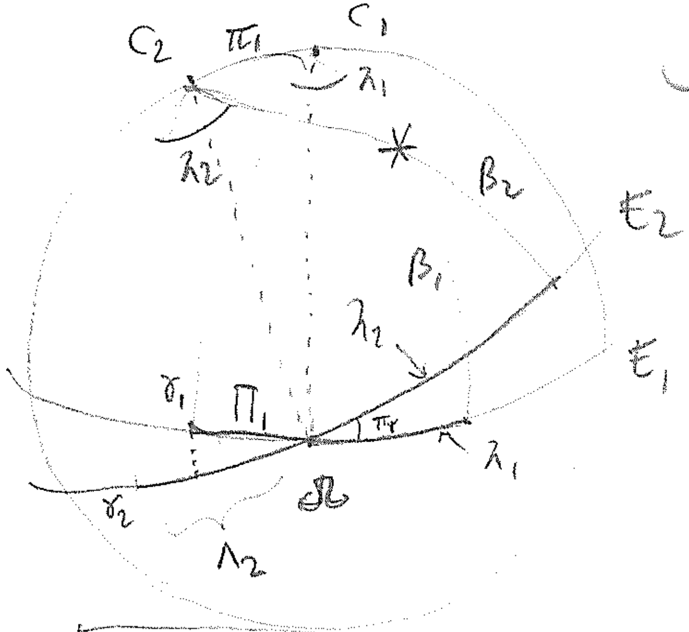
$$\sin(a-c) \cdot \sin A = \sin a \cos a \sin C' (\cos B - 1) + \sin a \cos C' \sin B =$$

$$= \frac{\sin 2a}{\cancel{2}} \sin C' \left(- \cancel{2} \sin^2 \frac{B}{2} \right) + \sin a \cos C' \sin B$$

PER CUI:

$$\sin(\psi_f - \psi) \sin \varepsilon = - \sin 2(\psi_f + \Omega) \sin \varepsilon_f \sin^2 \frac{i}{2} + \sin(\psi_f + \Omega) \sin i \cos \varepsilon_f$$

OK (12)



Passaggio da (α_1, δ_1) a (α_2, δ_2) con step intermedi passando da (β_1, λ_1) e (β_2, λ_2), cioè i passaggi sono: $(\alpha_1, \delta_1) \xrightarrow[1)]{} (\beta_1, \lambda_1) \xrightarrow[2)]{} (\beta_2, \lambda_2) \xrightarrow[3)]{} (\alpha_2, \delta_2)$

$$\boxed{\begin{array}{l} \Lambda_2 = \Pi_1 + \psi \\ \Pi_1 = \Omega, \quad \pi_1 = i \end{array}}$$

1) $(\alpha_1, \delta_1) \rightarrow (\beta_1, \lambda_1)$

$$\begin{cases} \cos\beta_1 \sin\lambda_1 = \sin\delta_1 \sin\varepsilon_1 + \cos\delta_1 \cos\varepsilon_1 \sin\alpha_1 \\ \cos\beta_1 \cos\lambda_1 = \cos\delta_1 \cos\alpha_1 \end{cases}$$

$$\sin\beta_1 = \sin\delta \cos\varepsilon_1 - \cos\delta_1 \sin\varepsilon_1 \sin\alpha_1$$

$$\begin{cases} \lambda_1 = \operatorname{atan}(\cos\beta_1 \sin\lambda_1, \cos\beta_1 \cos\lambda_1) \\ \beta_1 = \operatorname{asin}(\sin\beta_1) \end{cases}$$

2) $(\beta_1, \lambda_1) \rightarrow (\beta_2, \lambda_2)$

$$\begin{cases} \cos\beta_2 \sin(\lambda_2 - \Lambda_2) = \cos\beta_1 \sin(\lambda_1 - \Pi_1) \cos\pi_1 + \sin\beta_1 \sin\pi_1 \\ \cos\beta_2 \cos(\lambda_2 - \Lambda_2) = \cos\beta_1 \cos(\lambda_1 - \Pi_1) \end{cases}$$

$$\sin\beta_2 = \sin\beta_1 \cos\pi_1 - \cos\beta_1 \sin(\lambda_1 - \Pi_1) \sin\pi_1$$

con $\left.\begin{array}{l} \lambda_2 - \Lambda_2 = \lambda_2 - \Omega - \psi \quad e \quad \lambda_1 - \Pi_1 = \lambda_1 - \Omega \\ e \quad \pi_1 = i \end{array}\right\}$ da cui

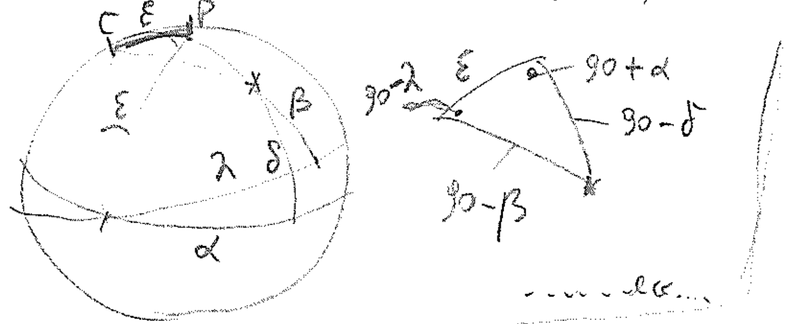
$$\begin{cases} \lambda_2 - \Omega - \psi = \operatorname{atan}\left[\cos\beta_1 \sin(\lambda_1-\Omega)\cos i + \sin\beta_1 \sin i,\ \cos\beta_1 \cos(\lambda_1 - \Omega)\right] \\ \beta_2 = \operatorname{asin}\left[\sin\beta_1 \cos i - \cos\beta_1 \sin(\lambda_1-\Omega) \sin i\right] \end{cases}$$

---

3) $(\beta_2, \lambda_2) \to (\alpha_2, \delta_2)$

$$\cos\delta_2 \sin\alpha_2 = \cos\beta_2 \cos\varepsilon_2 \sin\lambda_2 - \sin\beta_2 \sin\varepsilon_2$$
$$\cos\delta_2 \cos\alpha_2 = \cos\beta_2 \cos\lambda_2$$
$$\sin\delta_2 = \sin\beta_2 \cos\varepsilon_2 + \cos\beta_2 \sin\varepsilon_2 \sin\lambda_2$$

NOTA: Per ricordare le formule di trasformazione, usare i triangoli (da Smart)



...... ecc...

$\boxed{\text{Polo Celeste Nord}}$ $\alpha_1 = 180°$ $\delta_1 = 90°$

$$\begin{cases} \cos\beta_1 \sin\lambda_1 = \sin\varepsilon_1 \\ \cos\lambda_1 = 0 \\ \sin\beta_1 = \cos\varepsilon_1 \end{cases} \longrightarrow \boxed{\begin{array}{l} \lambda_1 = 90° \\ \beta_1 = \text{atan}\left(\cos\varepsilon_1, \sin\varepsilon_1\right) \rightarrow \beta_1 = 90 - \varepsilon_1 \end{array}}$$

$$\begin{cases} \lambda_2 - \Omega - \psi = \text{atan}\left[\sin(90-\Omega)\cos i + \cos\varepsilon_1 \sin i;\ \cos(90-\Omega)\right] \\ \beta_2 = \text{asin}\left[\cos\varepsilon_1 \cos i - \sin\varepsilon \sin(90-\Omega)\sin i\right] \end{cases}$$

ecc. ecc.